

Understanding Volatility of Indian and US Stock Markets in the Post Crisis Period

Abstract

This study analyses the volatility which exists in the Stock Market across the globe, the importance of measuring such volatility and the significance of accurate estimation of volatility of stock market in helping analysts take informed decision for trading, investing and hedging. In the current study, two stock indices representing their respective stock markets have been analysed. Of the two stock market indices chosen for the study, one is S&P BSE SENSEX, an index of the Indian Stock Market and the other is Dow Jones Industrial Average, an index of the US stock market. The model used for volatility estimation in the current study is the symmetric GARCH (1,1) model. The estimations show that the two indices exhibit almost same volatilities during the period under study, namely, January 2013 to December 2015.

Key words: Augmented Dickey Fuller test, DJIA, GARCH, SENSEX, Stock market, Unit root, Volatility

Introduction

The current study focuses on the measurement of Stock Market volatility with the help of GARCH Model. As volatility plays a real important role in determining the returns of any financial asset and how one can choose and select the instrument fitting their risk profile, its study has always excited the interest of researchers and analysts.

The present study is undertaken with an objective to measure the volatility of the returns of the chosen stock market indices using the univariate GARCH model. The indices chosen for study are Dow Jones Industrial Average (DJIA) and Bombay Stock Exchange (BSE) SENSEX. The two indices have been chosen based on the fact that they represent two countries that have important place in global economy.

The study is primarily based on secondary research having studied various research papers and trying to understand and analyse the insights drawn from approaches used for

Shalini Talwar

Associate Professor

Finance

K J Somaiya Institute of
Management Studies and
Research
Mumbai.

Vartica Khandelwal

PGDM Student

K J Somaiya Institute of
Management Studies and
Research
Mumbai.

measuring the volatility of various stocks and how it becomes an important tool for any financial professional to make any decision to buy or sell equity.

The authors have used the Augmented Dickey Fuller test to test the stationarity of the series at levels. The series was found to be stationary at first difference, as expected. Garch (1,1) model was applied to the log transformed returns for the two indices. The test results indicated that both US and Indian markets, as represented by the chosen indices, have almost the same volatility with Sensex showing a slightly higher volatility than Dow Jones.

The rest of the paper is organized as follows: literature review is discussed in the next section followed by data description, methodology, data analysis of the chosen stock market index, discussion of results and in the end, summary and conclusions.

Literature Review

Most financial time series exhibit a tendency of volatility clustering and mean reversion. Volatility clustering essentially means that variations of high magnitude tend to be followed by variations of large size, of either sign, and variations of small magnitude tend to be followed by variations of small size. Time series of stock market returns also exhibit the tendency of volatility clustering. There are various methods for quantifying and modelling this phenomenon. GARCH model is one of the methods of modelling this feature.

Generalised Autoregressive Conditional Heteroskadasticity (GARCH) was developed by Robert F, Engle, to estimate volatility in financial markets. This model is usually preferred by analysts who do financial modelling because it provides a more practical context than the other variants used to to predict the prices and rates of financial instruments in future. There are several forms of GARCH modelling available, with most basic one being GARCH (1, 1).

The general process for a GARCH model involves three-step approach. In the first step, a best-fitting autoregressive model is estimated. Thereafter, autocorrelations of the error terms are studied and finally the test for significance is applied.

Financial professionals use GARCH models for arriving at forecasts for trading, investing, hedging and dealing. Other than GARCH, analysts have also used the classic historical volatility method and the exponentially weighted moving average volatility method for estimating and predicting financial volatility but in the current study our focus would be on GARCH model only.

Recent developments in financial econometrics suggest the use of nonlinear time series structures to model the attitude of investors toward risk and expected return. When dealing with nonlinearities, one should be able to make a following distinction:

- Linear Time Series: shocks are assumed to be uncorrelated but not necessarily identically independently distributed.

- **Nonlinear Time Series:** shocks are assumed to be identically independent distributed, but there is a nonlinear function relating the observed time series and the underlying shocks or innovation or news.

Though earlier, volatilities were considered to be constant in the financial models, but now it is widely accepted and recognized that volatility varies over time. This recognition has led to an extensive research into the dynamic properties of stock market volatility. Stock volatility can be simply defined as a conditional variance or standard deviation of stock returns that is not directly observable. Volatility is been used as a proxy for riskiness associated with any financial asset. It is important to model and forecast conditional variance as accurately as possible because the optimal investment decision of investors depends on variance of returns that can change over time.

This model is useful when one wants to analyze and forecast volatility. Volatility forecasts are obtained from a variety of mean and variance specifications used in GARCH models which are then compared to a proxy of actual volatility calculated using daily data. It is usually done to serve three main purposes – risk management, optimal asset allocation and taking bets on future volatility. Though symmetric GARCH is used in this study, asymmetric GARCH models have been found to give better estimate than the symmetric one, confirming the presence of leverage effect. Volatilities cannot be directly measured, it can only be estimated in the context of the model applied. Research has confirmed the superiority of GARCH model over other models like historical average, exponentially weighted moving average as the GARCH model is able to replicate the fat tails observed in many of the high frequency asset return series, where large changes occur more than a normal distribution would imply.

Data Description

The daily closing prices of DJIA and S&P SENSEX have taken from their respective website for a period from January 2013 through December 2015 and used in the analysis. The details of indices with their brief description are exhibited in table 1.

INDEX	DESCRIPTION
DJIA	The Dow Jones Industrial Average is a price -weighted average of 30 blue-chip stocks that are generally the leaders in their industry. It has been a widely followed indicator of the stock market since October 1, 1928.
S&P BSE SENSEX	The S&P BSE Sensex Index is a cap -weighted index. The index members have been selected on the basis of liquidity, depth, and floating -stock-adjustment depth and industry representation. Sensex has a base date and value of 100 in 1978-1979. The Index shifted to free -float methodology since 09/01/03.

Source: <https://www.bloomberg.com>

Firstly, to understand the nature of the two indices under study, their descriptive statistics have been generated. Skewness, kurtosis and Jarque-Bera statistic thus generated help understand the behaviour of the markets under the study. Useful insights can be drawn from the descriptive statistics of the two indices exhibited in exhibits 1 and 2.

Exhibit 1: Descriptive Statistics of S&P BSE SENSEX

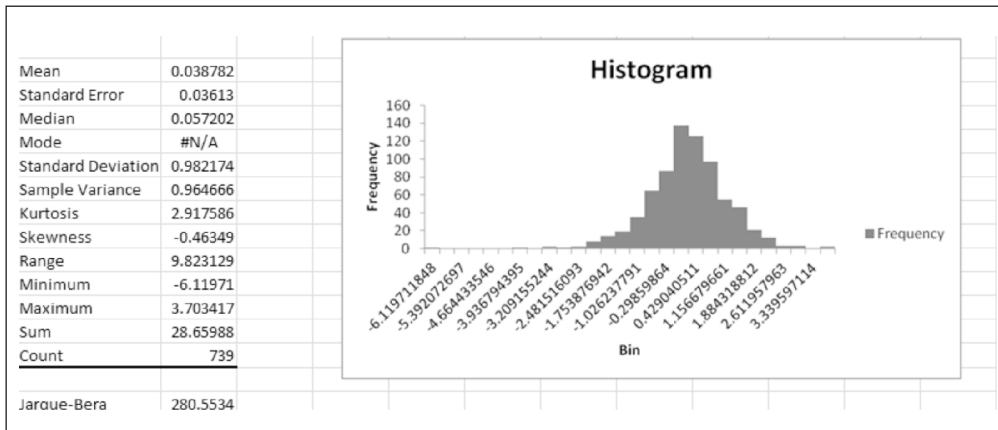
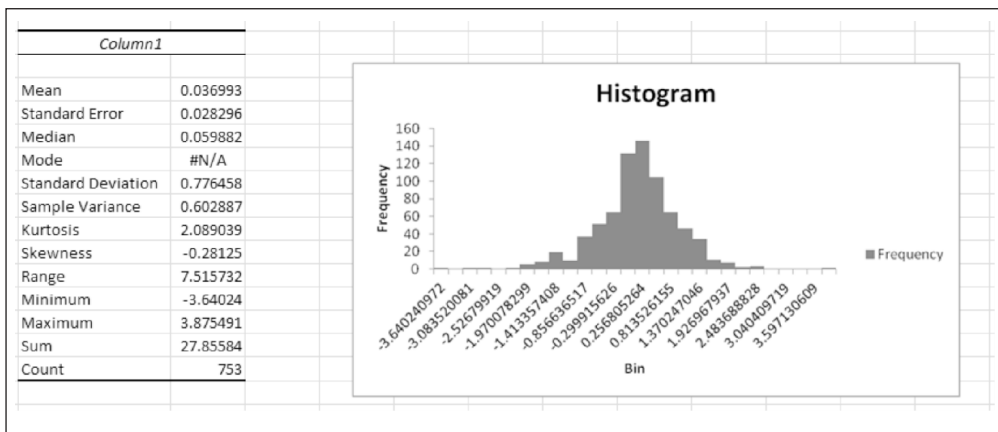


Exhibit 2: Descriptive Statistics of DJIA



The histograms clearly show that the distribution of daily returns are more tightly concentrated around zero for both indices.

The mean and standard deviation for DJIA daily returns are 0.03 and 0.77 respectively and the mean and standard deviation for SENSEX daily returns are 0.04 and 0.98 respectively Annualized, these values are approximately 7.56 and 12.22 for DJIA and 10.08 and 15.56 for SENSEX. SENSEX has a higher mean and volatility than DJIA. The lower volatility for the DJIA reflects that it is more diversified as there risk reduction due to diversification. The sample skewness for SENSEX, -0.4635, is slightly negative and it shows the approximate symmetry in the histogram in exhibit 1. The skewness for DJIA is moderately negative at -0.2812 which reflects slight siht in the histogram. The

kurtosis values for the two indices indicate that the tails of the histograms are not fat. This indicates less volatility in the indices during the period under the study.

Research Methodology

Since the data being used for this study is a time series data, it is important to test the stationarity of the series. Regression, if estimated using data with unit root as input, leads to spurious outcome that has no practical usefulness.

Augmented Dickey – Fuller test(ADF) is applied to check the stationarity of data. This test is well known test for unit root testing of time series data. If the Y_t is the time series to be tested for unit root, then the test statistic for the test is given by the below relationship:

$$\Delta y_t = \rho y_{t-1} + \mu + \lambda t + \alpha_i \sum_{i=1}^n y_{t-i} + u_t$$

Volatility, the importance of quantifying it and what place it holds in various financial markets have been discussed in detail in the literature review section. Economic tools such as GARCH (1,1) has been widely used to estimate volatility because of its various advantages over other well known techniques like historical average, exponentially weighted moving average.

The GARCH model proposed by Bollerslev (1986), is the extension of the ARCH model. This model is based on the assumption that the conditional variance is dependent on its previous own lags. The equation thus follows:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2$$

Where the parameter constraints $\alpha_i > 0$ ($i = 0, 1, \dots, q$) and $\beta_j > 0$ ($j = 1, \dots, p$) assure that $\sigma_t^2 > 0$.

The above represents the basis GARCH(p,q) model. The unconditional variance is computed as follows:

$$\bar{\sigma}^2 = \text{var}(\varepsilon_t) = \frac{\alpha_0}{1 - (\sum_{i=1}^q \alpha_i + \sum_{j=1}^p \beta_j)}$$

In the present study, the authors have used GARCH(1,1) to estimate the voliality of the two indices under the study . In practice, this model takes only three parameters as specified in the equation of conditional variance which is sufficient to capture the volatility clustering in the data. The conditional variance equation of this model becomes:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

and the conditional variance is $\bar{\sigma}^2 = \alpha_0 / (1 - \alpha_1 - \beta_1)$.

Data Analysis

Analysis of ADF Test Output

The chosen indices have been tested for stationarity using ADF. Both the series were found to be non-stationary at levels and stationary at first difference. This is interpreted using the probability of the test statistic. The results of ADF for both, SENSEX and DJIA is given in table 2 & 3. The output show that SENSEX is non-stationary at level as the p-value > significance value which means we can't reject the Null hypothesis (H_0) which states that the variable has a unit root. At first difference or log transformed returns, the p-value < significance value which means we can reject the Null hypothesis (H_0) which states that the variable has a unit root. For DJIA also, the output reflects the same results.

Table 2: ADF Results for SENSEX

Sensex		Sensex returns	
Output		Output	
Dickey Fuller Test Statistic	0.660554	Dickey Fuller Test Statistic	-9.2009
p-Value	0.99	p-Value	0.01
LagOrder	9	LagOrder	9

Table 3: ADF Results for DJIA

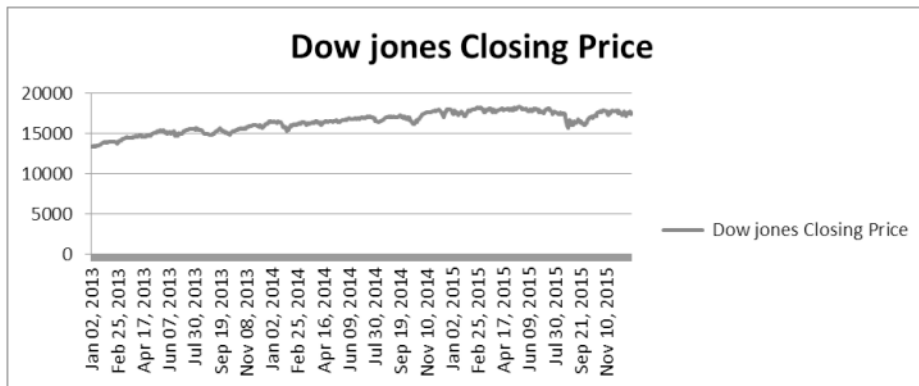
Dow Jones		Dow Jones Returns	
Output		Output	
Dickey Fuller Test Statistic	-2.89094	Dickey Fuller Test Statistic	-9.45527
p-Value	0.201108	p-Value	0.01
LagOrder	9	LagOrder	9

The same results are confirmed by plotting the time series of the two indices at levels and returns. The plots of level exhibits non-stationarity for both indices whereas the plots of returns shows stationarity as shown in exhibits 3 through 6.

Exhibit 3: Time Series Plot of SENSEX at Level



Exhibit 4: Time Series Plot of DJIA at Level



From the above, we see on the whole the market has shown an upward trend. Indian market indicator, Sensex in 2013 and 2014 was on a little downside and then it picked up in Mid June 2014 whereas DJIA was on a upside trend since 2013.

Exhibit 5: Time Series Plot of SENSEX at First Difference

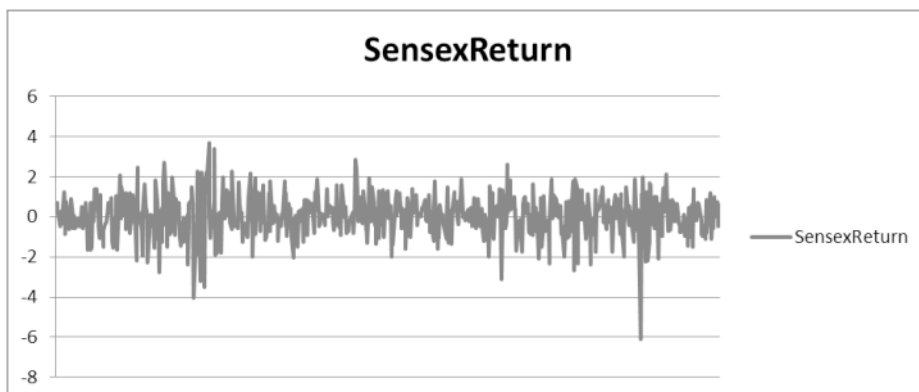
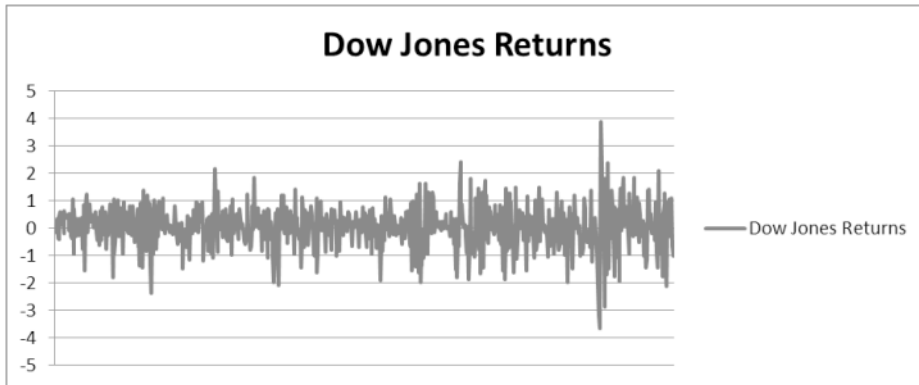


Exhibit 6: Time Series Plot of DJIA at First Difference



There is a sharp drop seen in the Sensex returns. This was on September 2015 when China devalued its currency and there was a whole drop seen in the overall global markets. Lot of volatility seen in the latter half of year 2015 due to the slowdown in China.

Estimation Output of GARCH

The output of the estimation of volatility of the indices through GARCH(1,1) model is given in tables 4 and 5.

Table 4 : GARCH (1,1) Output for SENSEX

Date	Sensex Closing Price	Sensex Return	Variance	Likelihood
01-Jan-13	19580.81			
02-Jan-13	19714.24	0.006814325		
03-Jan-13	19764.78	0.002563629	0.000046435	9.835921
04-Jan-13	19784.08	0.000976484	9.94652120E-06	11.42242
07-Jan-13	19691.42	-0.00468356	9.94330502E-06	9.312527
21-Dec-15	25735.9	0.008490855	1.00123716E-05	4.311135
22-Dec-15	25590.65	-0.00564387	9.98402674E-06	8.324104
23-Dec-15	25850.3	0.010146284	9.96099226E-06	1.181812
24-Dec-15	25838.71	-0.00044835	1.00016869E-05	11.49266
28-Dec-15	26034.13	0.007563071	9.94287429E-06	5.765786
29-Dec-15	26079.48	0.001741944	9.97550097E-06	11.2112
30-Dec-15	25960.03	-0.00458023	9.94449612E-06	9.408932
31-Dec-15	26117.54	0.006067404	9.95476748E-06	7.819392
Trial values	0.994275922	9.94276E-06	ω	1409.848
	0	0	β	
	0.005724078	0.000572408	α	
Long term variance per day		9.94845E-06		
Long term volatility per day		0.31541170%		

Table 5 : GARCH (1,1) Output for DJIA

Date	Dow Jones Closing Price	Dow Jones Return	Variance	Likelihood
Jan 02, 2013	13412.55			
Jan 03, 2013	13391.36	-0.001579864		
Jan 04, 2013	13435.21	0.003274499	0.000002496	8.60496854
Jan 07, 2013	13384.29	-0.003790041	9.50904706E-06	10.052662
Dec 21, 2015	17251.62	0.00718508	1.18735981E-05	6.99328014
Dec 22, 2015	17417.27	0.009601997	9.73397973E-06	2.06808431
Dec 23, 2015	17602.61	0.010641162	9.95709736E-06	0.14500123
Dec 24, 2015	17552.17	-0.002865484	1.00727779E-05	10.6905067
Dec 28, 2015	17528.27	-0.001361655	9.49523674E-06	11.3694535
Dec 29, 2015	17720.98	0.01099424	9.46027909E-06	-1.2085193
Dec 30, 2015	17603.87	-0.006608551	1.01147858E-05	7.1837791
Dec 31, 2015	17425.03	-0.01015913	9.69024799E-06	0.89369119
Trial values	0.945008306	9.45008E-06 ω		4181.11072
	0	0 β		
	0.054991694	0.005499169 α		
	1			
Long term variance per day		9.50234E-06		
Long term volatility per day		0.30825863%		

The volatilities obtained by using GARCH(1,1) per day for the Sensex and Dow Jones are:

Sensex: 0.315%

Dow Jones: 0.308%

We observe that both the markets have almost the same volatility with Sensex showing a slightly higher volatile than Dow Jones. Just to better understand the movement between the two indices under the study, the authors generated the correlation matrix and found that the two have a very low degree of negative correlation. The matrix is exhibited in table 6.

Table 6: Correlation Matrix for SENSEX and DJIA

	<i>SENSEX</i>	<i>DJIA</i>
SENSEX	1	
DJIA	-0.02146	1

Summary and Conclusion

Measurement of stock market volatility is important for several reasons. Detection of various volatility trends would help provide insight for professionals to design superior investment strategies and design better performing portfolios. Accurate forecasts of stock market volatility also may help improve the performance of various option pricing models. Thus, volatility estimation becomes an important and essential part in most finance decisions be it derivative pricing or asset allocation.

During the three-year period under the study, both US and Indian equity markets as represented by the selected indices have displayed almost equal amount of volatility. Volatility of markets can come from various factors. The key factors that impacted the global equity markets during the time period under the study were China slow down, Speculation over Federal Reserve raising the interest rates, uncertainty over Greece, spate of terrorist attacks and slow global economic recovery. Measures should be taken towards to reduce the volatile to as low as possible so that the returns expected by the investors are not affected much.

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